

Mathematics Department

Preparation for Studying A-level Mathematics: Additional
Notes

Introduction

You will have been provided with a booklet containing examples and questions to help you prepare for studying A-level Mathematics. This booklet is to help provide you with some extra support and notes to help you complete that booklet. If there are any questions you have regarding anything in either booklet, make a note of it and make sure you ask your teacher in the first week. You will be assessed in the first half of the Autumn Term, and that assessment will involve knowledge in the preparation booklet.

Dr. J. Samuel
Head of Mathematics

Acknowledgements:

This booklet is based on material originally produced by Mr. D. Jeffers, who was a fantastic Head of Mathematics.

Surds

General Definitions

A **Surd** is an irrational square root. The value of a surd cannot be written down exactly; it has an infinite number of decimal places that never recur.

Any square root whose value is not an integer is a surd. E.g. $\sqrt{16}$, $\sqrt{25}$ are **not** surds but $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ are surds.

Essential Knowledge

You need to be able to simplify and manipulate surds.

You **can** multiply surds i.e. $\sqrt{3} \times \sqrt{5} = \sqrt{15}$ but you **cannot** add them i.e. $\sqrt{3} + \sqrt{5} \neq \sqrt{8}$.

If the surds are the same you can combine them i.e. $2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$.

You can think of surds as acting like x 's and y 's in normal algebra and combine terms that contain the same surds. i.e. $\sqrt{5} + 5\sqrt{7} + 3\sqrt{5} - 2\sqrt{7} = 4\sqrt{5} + 3\sqrt{7}$.

To simplify a surd such as $\sqrt{12}$

- Look for a square factor of the number under the square root.
- Write the number under the square root as a product of its square factor and another number.
- Square root the square number and leave the other number alone.

So $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$ in a simplified form and $\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$.

You need to be able to multiply brackets and simplify. This is very similar to the work you have done multiplying out quadratic brackets.

$$\begin{aligned}(\sqrt{3} + 2\sqrt{5})(4\sqrt{3} + 3\sqrt{5}) &= \sqrt{3} \times 4\sqrt{3} + \sqrt{3} \times 3\sqrt{5} + 2\sqrt{5} \times 4\sqrt{3} + 2\sqrt{5} \times 3\sqrt{5} \\ &= 4\sqrt{9} + 3\sqrt{15} + 8\sqrt{15} + 6\sqrt{25} \\ &= 12 + 3\sqrt{15} + 8\sqrt{15} + 30 \\ &= 32 + 11\sqrt{15}\end{aligned}$$

You need to be able to **Rationalise the Denominator**. This means that you write a fraction with no surds in the denominator. These come in two types.

“Rationalise the denominator of the fraction $\frac{2}{\sqrt{7}}$ means “get rid of the square root on the bottom”.

You should have seen by now that multiplying a surd by itself gets rid of the root sign, that is $\sqrt{7} \times \sqrt{7} = 7$. You should also know from your work on fractions that we are allowed to multiply the top and the bottom of a fraction by the same thing without changing the size of the fraction. Put these two facts together and you've got the answer.

$$\frac{2}{\sqrt{7}} = \frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$$

The denominator has been rationalised.

The second type is more difficult and appears on GCSE papers quite often. This type involves rationalising the denominator of fractions such as $\frac{2}{\sqrt{3} + \sqrt{5}}$.

The rule here is to change the sign in the middle of the bit on the bottom from + to - or from - to + and then multiply the top and bottom by the new expression.

$$\frac{2}{\sqrt{3} + \sqrt{5}} = \frac{2}{(\sqrt{3} + \sqrt{5})} \times \frac{(\sqrt{3} - \sqrt{5})}{(\sqrt{3} - \sqrt{5})} = \frac{2\sqrt{3} - 2\sqrt{5}}{3 - \sqrt{15} + \sqrt{15} - 5} = \frac{2\sqrt{3} - 2\sqrt{5}}{-2} = \sqrt{5} - \sqrt{3}$$

You might need to look carefully at that last stage, both of the terms in the numerator have been divided by -2.

Typical Examination Questions

Question Rationalise the denominator of $\frac{21}{\sqrt{7}}$ and simplify your answer.

Solution

$$\frac{21}{\sqrt{7}} = \frac{21}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{21\sqrt{7}}{7} = 3\sqrt{7}$$

Question Expand $(\sqrt{5} + 2\sqrt{3})(\sqrt{5} - 2\sqrt{3})$ expressing your answer as simply as possible.

Solution

$$(\sqrt{5} + 2\sqrt{3})(\sqrt{5} - 2\sqrt{3}) = 5 - 2\sqrt{15} + 2\sqrt{15} - 4 \times 3 = -7$$

Question Work out $\frac{(5 + \sqrt{3})(5 - \sqrt{3})}{\sqrt{22}}$ giving your answer in its simplest form.

Solution

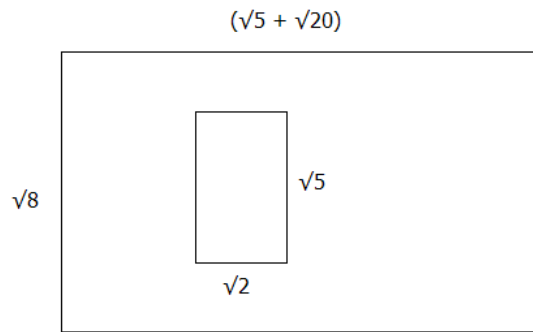
$$\frac{(5 + \sqrt{3})(5 - \sqrt{3})}{\sqrt{22}} = \frac{25 - 5\sqrt{3} + 5\sqrt{3} - 3}{\sqrt{22}} = \frac{22}{\sqrt{22}} = \sqrt{22}$$

Question Given that $\sqrt{40} = k\sqrt{10}$, find the value of k .

Solution

$$\sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10} \Rightarrow k = 2$$

Question



A large rectangular piece of card is $(\sqrt{5} + \sqrt{20})$ cm long and $\sqrt{8}$ cm wide.

A small rectangle $\sqrt{2}$ cm long and $\sqrt{5}$ cm wide is cut out of the piece of card.

Express the area of the card that is left as a percentage of the area of the large rectangle.

Solution

$$\begin{aligned}\text{Area of the large rectangle} &= \sqrt{8}(\sqrt{5} + \sqrt{20}) \\ &= \sqrt{40} + \sqrt{160} \\ &= \sqrt{4 \times 10} + \sqrt{16 \times 10} \\ &= 2\sqrt{10} + 4\sqrt{10} = 6\sqrt{10}\end{aligned}$$

$$\text{Area of the small rectangle} = \sqrt{2} \times \sqrt{5} = \sqrt{10}$$

$$\text{Area of card that is left} = 6\sqrt{10} - \sqrt{10} = 5\sqrt{10}$$

$$\text{Required percentage} = \frac{5\sqrt{10}}{6\sqrt{10}} \times 100\% = \frac{500}{6}\% = 83.3\% \text{ to 3sf.}$$

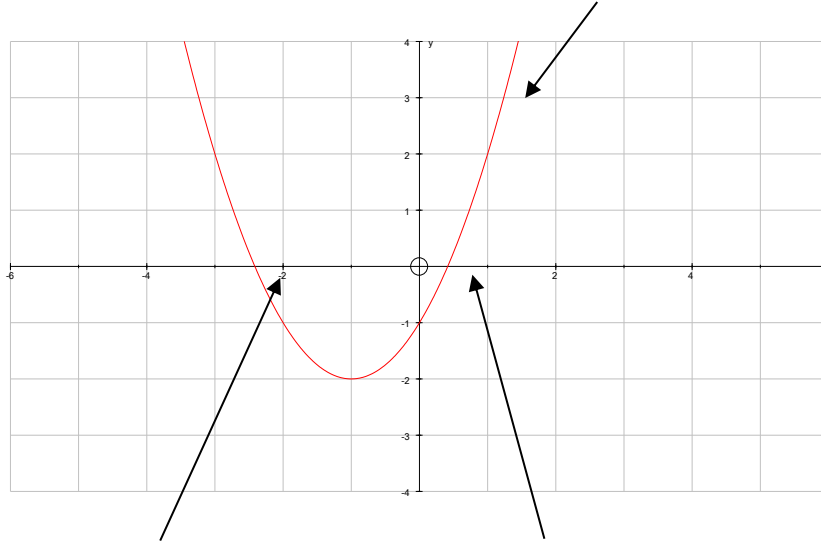
The Quadratic Formula

General Definitions

A quadratic is an equation that contains a variable (a letter) that has been squared.

$x^2 + 2x - 3 = 0$, $3x^2 - 4x + 1 = 0$, $2x^2 - 7 = 0$ and $x^2 + 2x = 0$ are all quadratic equations.

The graph of a quadratic equation is a curve called a parabola.



When you solve a quadratic equation you are finding the x-coordinates of the points where the graph of the equation crosses the x-axis.

You have seen how to solve a quadratic equation using factorisation but if the equation will not factorise we need to use the **Quadratic Formula**.

Essential Knowledge

The quadratic formula is based on the general quadratic equation

$$ax^2 + bx + c = 0$$

where a , b and c are numbers that you need to identify (more in a minute).

When you have found the values of a , b and c you need to substitute them into the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula needs some care when using it.

We will solve the equation $x^2 + 2x - 7 = 0$ using this formula.

The value of a is always the number in front of the x^2 term (called the coefficient of x^2). In this case there is no number there so its value is 1.

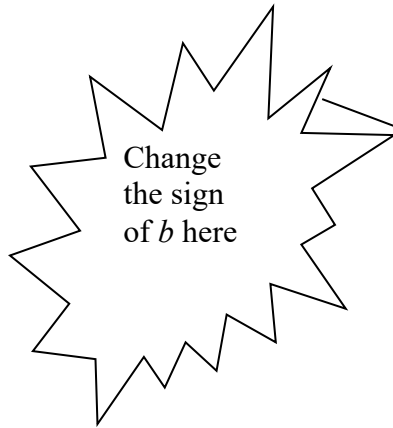
The value of b is always the number in front of the x term (called the coefficient of x). In this case the value of b is 2.

The value of c is always the number that is on its own (called the constant). In this case the value of c is -7.

So to summarise,

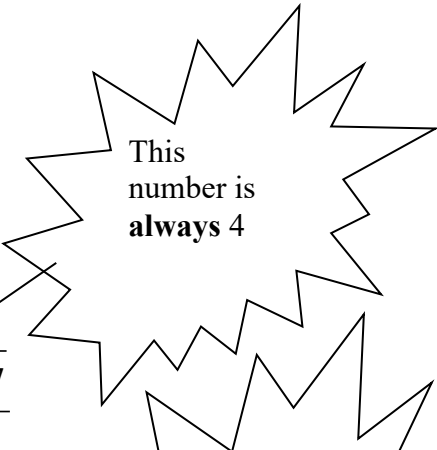
$$a=1, b=2, c=-7$$

We substitute these values into the formula,

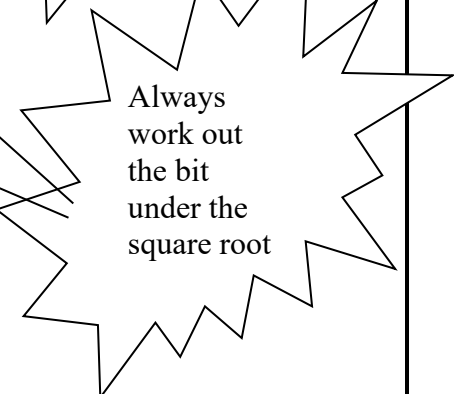


$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times -7}}{2 \times 1}$$

$$\Rightarrow x = \frac{-2 \pm \sqrt{32}}{2}$$



This number is always 4



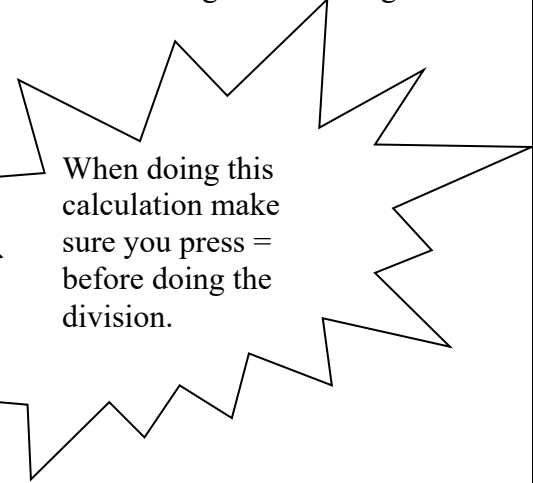
Always work out the bit under the square root

NB – at GCSE, if the number under the square root is negative you have made a mistake. Go back and check your working if this happens.

To finish the problem you need to know the meaning of the \pm symbol. As you saw in the graph on the first page there are usually two points where the graph crosses the x -axis. These two solutions are found by using the plus sign first and then repeating the calculation using the minus sign. So the two solutions are,

$$x = \frac{-2 + \sqrt{32}}{2} \text{ and } x = \frac{-2 - \sqrt{32}}{2}$$

$$\Rightarrow x = 1.83 \text{ or } x = -3.83$$



When doing this calculation make sure you press = before doing the division.

You need to take special care if the value of b is negative.

To solve the equation $x^2 - 7x + 2 = 0$ start by identifying the values of a , b and c as before. Write these values down at the start of your solution.

$$a = 1, b = -7, c = 2$$

Now substitute these values into the formula.

Change the
sign of b
from - to +

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4 \times 1 \times 2}}{2 \times 1}$$

Notice that the value of b has been put into brackets because it is negative. **You must do this on your calculator as well or it will give you the wrong answer.** Do the calculation under the square root sign yourself and make sure you get the same answer as the one below.

$$x = \frac{7 \pm \sqrt{41}}{2}$$

$$\Rightarrow x = \frac{7 + \sqrt{41}}{2} \text{ or } x = \frac{7 - \sqrt{41}}{2}$$

$$\Rightarrow x = 6.70 \text{ or } x = 0.298$$

Finish the calculation yourself and make sure that you get the same answers.

The final stage is to consider the situation when the coefficient of x^2 is greater than 1.

We solve the equation $2x^2 + x - 4 = 0$ giving the answers correct to 3 s.f.

For this equation $a = 2, b = 1, c = -4$

Substituting into the formula gives

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 2 \times -4}}{2 \times 2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{33}}{4}$$

$$\Rightarrow x = \frac{-1 + \sqrt{33}}{4} \text{ or } x = \frac{-1 - \sqrt{33}}{4}$$

$$\Rightarrow x = 1.19 \text{ or } x = -1.39$$

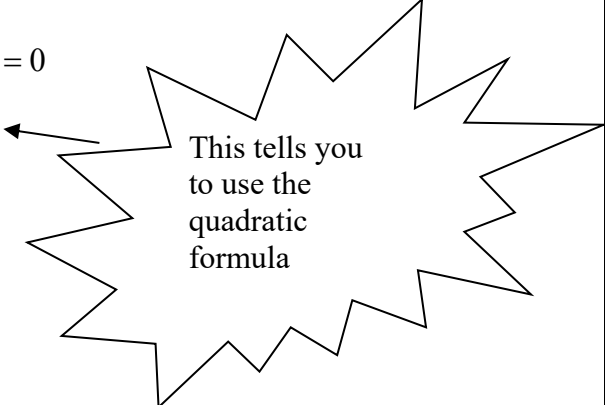
Once again, work through this solution and make sure that you can get the same answers.

Typical Examination Questions

Question Solve this quadratic equation.

$$x^2 - 5x - 8 = 0$$

Give your answers correct to 3 significant figures.



This tells you
to use the
quadratic
formula

Solution $a=1, b=-5, c=-8$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 1 \times -8}}{2 \times 1}$$

$$\Rightarrow x = \frac{5 \pm \sqrt{57}}{2}$$

$$\Rightarrow x = \frac{5 + \sqrt{57}}{2} \text{ or } x = \frac{5 - \sqrt{57}}{2}$$

$$\Rightarrow x = 6.27 \text{ or } x = -1.27$$

Other examination questions involve an extra problem solving stage. These will be covered by a later note.

Completing the Square

General Definitions

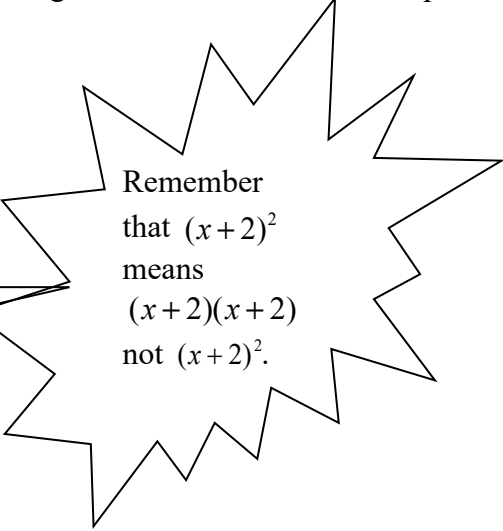
There are no real general definitions for this topic. It is a skill that allows us to analyse quadratic functions a little more but it may seem a little nonsensical to some.

Just learn the process and the results it offers. At best it does allow us to prove the quadratic formula, which I will do at the end of these notes, but for now...

Essential Knowledge

Consider the expression $(x+2)^2 - 3$. The only thing we can do with this is to expand the brackets and simplify it.

$$\begin{aligned}(x+2)^2 - 3 \\ &= (x+2)(x+2) - 3 \\ &= x^2 + 2x + 2x + 4 - 3 \\ &= x^2 + 4x + 1\end{aligned}$$



Remember
that $(x+2)^2$
means
 $(x+2)(x+2)$
not $(x+2)^2$.

The process of **Completing the Square** is the reverse of the process above, that is, we start with $x^2 + 4x + 1$ and turn it into $(x+2)^2 - 3$.

Put half the coefficient of x into the bracket.

$$x^2 + 4x + 1 = (x+2)^2 - 4 + 1$$

Subtract the number in the bracket squared.

Then simplify the constants on the end,

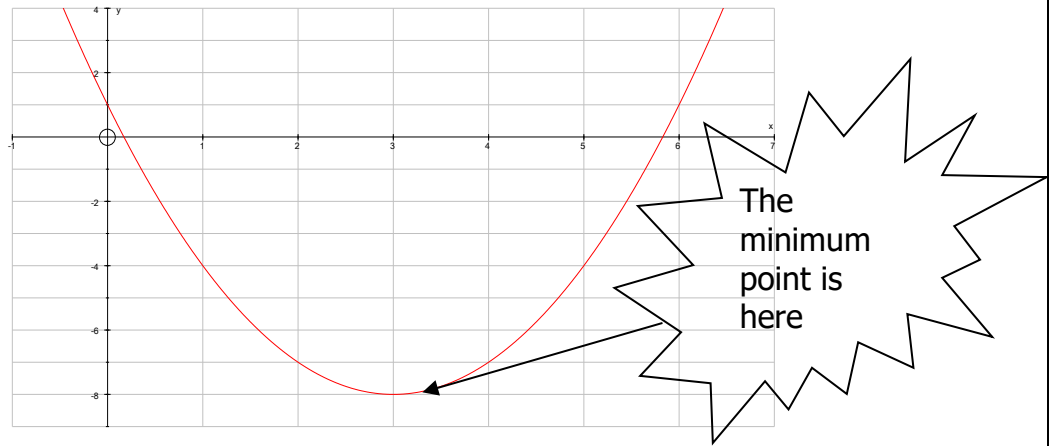
$$\Rightarrow x^2 + 4x + 1 = (x+2)^2 - 3$$

which is what we started off with.

So, in response to the question to write $x^2 - 6x + 1$ in the form $(x - a)^2 - b$ you would do the following.

$$x^2 - 6x + 1 = (x - 3)^2 - 9 + 1 = (x - 3)^2 - 8$$

We can use this form for the quadratic expression to find the minimum point of its graph.



As you can see from the diagram, the minimum point of this parabola is at the point (3, -8).

This point can be identified from $(x - 3)^2 - 8$ firstly by working out the x value where the bracket is zero, that is, when $x - 3 = 0$. This happens when $x = 3$ so the x coordinate of the point is 3. The number on the end, in this case -8, is the y coordinate of the minimum point giving the answer shown above, which is (3, -8).

Problem Find the coordinates of the minimum point of the graph $y = x^2 + 8x - 1$.

Solution

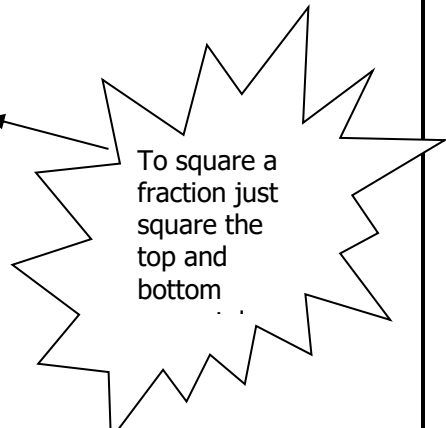
$$y = (x + 4)^2 - 16 - 1$$
$$\Rightarrow y = (x + 4)^2 - 17$$

Now $x + 4 = 0 \Rightarrow x = -4$, so the coordinates of the minimum point are (-4, -17).

If the coefficient of x is odd the process is the same but you have to take a little care over the working.

To write the expression $x^2 - 3x + 2$ in the form $(x - a)^2 - b$ put half the coefficient of x into the bracket as before but leave it in the form $\frac{3}{2}$ rather than 1.5 or $1\frac{1}{2}$ because it is easier to square when in this form.

$$\begin{aligned}
 x^2 - 3x + 2 &= \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + 2 \\
 \Rightarrow x^2 - 3x + 2 &= \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{8}{4} \\
 \Rightarrow x^2 - 3x + 2 &= \left(x - \frac{3}{2}\right)^2 - \frac{1}{4}
 \end{aligned}$$

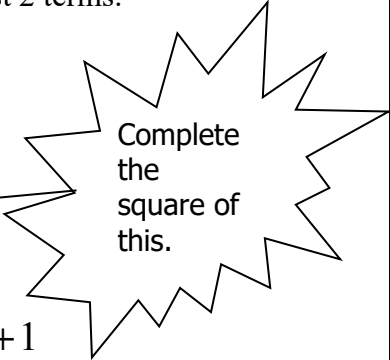


To square a fraction just square the top and bottom.

If the coefficient is not 1 the process is slightly different. This type of problem does sometimes turn up on GCSE papers.

To complete the square of $2x^2 - 6x + 1$ firstly factorise 2 out of the first 2 terms.

$$\begin{aligned}
 2x^2 - 6x + 1 &= 2(x^2 - 3x) + 1 \\
 \Rightarrow 2x^2 - 6x + 1 &= 2\left(\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right) + 1
 \end{aligned}$$



Complete the square of this.

Now multiply the bracket out and simplify.

$$\begin{aligned}
 \Rightarrow 2x^2 - 6x + 1 &= 2\left(x - \frac{3}{2}\right)^2 - \frac{9}{2} + 1 \\
 \Rightarrow 2x^2 - 6x + 1 &= 2\left(x - \frac{3}{2}\right)^2 - \frac{9}{2} + \frac{2}{2} \\
 \Rightarrow 2x^2 - 6x + 1 &= 2\left(x - \frac{3}{2}\right)^2 - \frac{7}{2}
 \end{aligned}$$

So the minimum point of this expression would be $\left(\frac{3}{2}, -\frac{7}{2}\right)$

Finally, a proof of the quadratic formula using the techniques above.

$$\begin{aligned}ax^2 + bx + c &= 0 \\ \Rightarrow a\left(x^2 + \frac{b}{a}x\right) + c &= 0 \\ \Rightarrow a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right) + c &= 0 \\ \Rightarrow \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} &= -\frac{c}{a} \\ \Rightarrow \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2}{4a^2} - \frac{4ac}{4a^2} \\ \Rightarrow \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\ \Rightarrow x + \frac{b}{2a} &= \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \\ \Rightarrow x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\end{aligned}$$

Those of you who want an A*, have a good look at the proof above. Even if you are not asked to prove it in the exam the process of understanding it will do you good.

Typical Examination Questions

Question The expression $x^2 - 6x + 14$ can be written in the form $(x - p)^2 + q$ for all values of x . Find the values of p and q .

Solution

$$\begin{aligned}x^2 - 6x + 14 &= (x - 3)^2 - 9 + 14 \\ \Rightarrow x^2 - 6x + 14 &= (x - 3)^2 + 5 \\ \Rightarrow p &= 3 \text{ and } q = 5\end{aligned}$$

The next question came up on the 2003 GCSE calculator paper and looks more difficult than the one above because of the algebraic coefficients. It is, in fact easier. Look carefully at this solution and don't be scared by the wording of a question.

Question For all values of x and m , $x^2 - 2mx = (x - m)^2 - k$

(a) Express k in terms of m .

The expression $x^2 - 2mx$ has a minimum value as x varies.

(b) (i) Find the minimum value of $x^2 - 2mx$. Give your answer in terms of m .

(ii) State the value of x for which this minimum value occurs. Give your answer in terms of m .

Solution

$$x^2 - 2mx = (x - m)^2 - m^2$$

The coefficient of x is $-2m$ so half of this, $-m$, goes inside the bracket. You then subtract m^2 at the end.

$$\Rightarrow k = m^2$$

The minimum value of this is the y coordinate which is $-m^2$. The value of the bracket will be zero when $x = m$ so this is the answer to (b) (ii).

Algebra - Indices

General Rules

In the rules below, a , b , m and n are integers.

$$x^a \times x^b = x^{a+b}$$

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

$$x^a \div x^b = x^{a-b}$$

$$x^{\frac{1}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

$$(x^a)^b = x^{a \times b}$$

$$x^0 = 1$$

$$x^{-a} = \frac{1}{x^a}$$

Essential Knowledge

You need to ensure that you know your square numbers, your cube numbers and powers of 2. Although not essential, you will also find it useful to know your powers of 3. All of this knowledge is given below.

Square Numbers:

$$1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25, 6^2 = 36, 7^2 = 49, 8^2 = 64, 9^2 = 81, 10^2 = 100, 11^2 = 121, 12^2 = 144, \\ 13^2 = 169, 14^2 = 196, 15^2 = 225, 16^2 = 256$$

Cube Numbers:

$$1^3 = 1, 2^3 = 8, 3^3 = 27, 4^3 = 64, 5^3 = 125, 6^3 = 216, 7^3 = 343, 8^3 = 512, 9^3 = 729, 10^3 = 1000$$

Powers of 2:

$$2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32, 2^6 = 64, 2^8 = 128, 2^9 = 256, 2^{10} = 512$$

Powers of 3:

$$3^0 = 1, 3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243, 3^6 = 729$$

You should also be able express one number as a power of another number, for example:

$$8^4 = (2^3)^4 \\ = 2^{12}$$

$$9^4 = (3^2)^4 \\ = 3^8$$

You may need to re-write the re-write the question, for example write the mixed number as a top heavy fraction or write the decimal as a mixed number.

Remember the question could be phrased in such a manner that requires you to show all steps of your working out, you will not simply be able to use the calculator to gain full credit. The examiner is looking for you to fully demonstrate your understanding.

Typical Examination Questions

You should be able to do the questions below without a calculator.

Question Find the value of:

(a) 9^{-2}

(b) $27^{\frac{1}{3}}$

(c) $\left(\frac{1}{2}\right)^{-2}$

Answer (a) $9^{-2} = \frac{1}{9^2} = \frac{1}{81}$

(b) $27^{\frac{1}{3}} = 3$

This is a fact that should know.

(c)

$$\begin{aligned}\left(\frac{1}{2}\right)^{-2} &= (2^{-1})^{-2} \\ &= 2^2 \\ &= 4\end{aligned}$$

Question Find the value of

$$4^{\frac{1}{4}} \times 2^3 \times 4^{\frac{3}{4}}.$$

Answer Since $4^{\frac{1}{2}} = 2$, then $4^{\frac{1}{4}} = (2^2)^{\frac{1}{4}} = 2^{\frac{2}{4}} = 2^{\frac{1}{2}} = \sqrt{2}$.

Using the fact above, we obtain,

$$\begin{aligned}4^{\frac{1}{4}} \times 2^3 \times 4^{\frac{3}{4}} &= 2^{\frac{1}{2}} \times 2^3 \times 2^{\frac{3}{2}} \\ &= 2^{\frac{1}{2}+3+\frac{3}{2}} \\ &= 2^5 \\ &= 32\end{aligned}$$

Question Solve the equation

$$3^{(2x-1)} = 9^{-x}.$$

Answer First re-write everything as powers of 3.

$$3^{(2x-1)} = (3^2)^{-x}$$

Now we can equate the powers and solve the equation.

$$2x - 1 = -2x$$

$$4x - 1 = 0$$

$$4x = 1$$

$$x = \frac{1}{4}$$

Question Solve the equation

$$4(2^{x^2}) = 8^x.$$

Answer First re-write everything as powers of 2.

$$2^2 \times (2^{x^2}) = (2^3)^x$$

Now we can equate powers and solve the equation.

$$2 + x^2 = 3x$$

$$x^2 - 3x + 2 = 0$$

$$(x - 2)(x - 1) = 0$$

$$x = 2 \text{ or } x = 1$$

Algebra – Simultaneous Equations

Essential Knowledge

You will need to be able add and subtract equations.

You will also need to be able to substitute from one equation into another equation.

The questions in this section can be completed without a calculator, and you will be expected to do so, depending on how the question has been worded.

Remember that when solving simultaneous equations you finding where the two straight lines intersect (if the equations are linear), or where the curve and line intersect (if one equation is a quadratic and one equation is linear).

Typical Examination Questions

You should be able to do the questions below without a calculator.

Question Solve the simultaneous equations

$$2x + 3y = 7$$

$$5x + 2y = 23$$

Answer You need to equalize the ys, multiply the first equation by 2 and multiply the second equation by 3.

$$4x + 6y = 14$$

$$15x + 6y = 69$$

Subtracting:

$$-11x = -55$$

$$x = 5$$

Substituting into the first equation gives:

$$10 + 3y = 7$$

$$3y = -3$$

$$y = -1$$

This means that the two lines intersect at the point (5,-1).

You should also substitute your answers in both equations to check that both equations are satisfied by the answer you have obtained.

Question Solve the simultaneous equations:

$$\begin{aligned}x - 2y &= 7 \\ x^2 + 4y^2 &= 37\end{aligned}$$

Answer Re-arranging the first equation gives $x = 7 + 2y$.

Substituting into the second equation gives

$$\begin{aligned}(7 + 2y)^2 + 4y^2 &= 37 \\ 49 + 28y + 4y^2 + 4y^2 - 37 &= 37 \\ 8y^2 + 28y + 12 &= 0\end{aligned}$$

Divide by 4:

$$\begin{aligned}2y^2 + 7y + 3 &= 0 \\ (2y + 1)(y + 3) &= 0\end{aligned}$$

So either: $2y + 1 = 0 \Rightarrow y = -\frac{1}{2}$

or $y + 3 = 0 \Rightarrow y = -3$

When $y = -\frac{1}{2}, x = 7 - 1 = 6$.

When $y = -3, x = 7 - 6 = 1$.

So the solutions are:

$$\left\{ \begin{array}{l} x = 6 \\ y = -\frac{1}{2} \end{array} \right\} \text{ and } \left\{ \begin{array}{l} x = 1 \\ y = -3 \end{array} \right\}$$

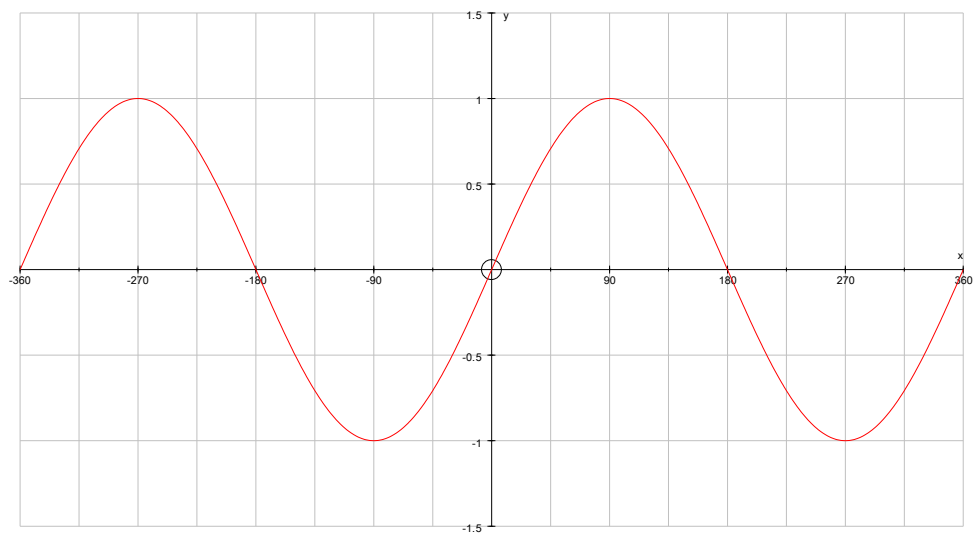
You must make sure that you pair up your solutions correctly, that is, you correctly state which value of x is pair with which value of y .

Graphs of Trig Functions

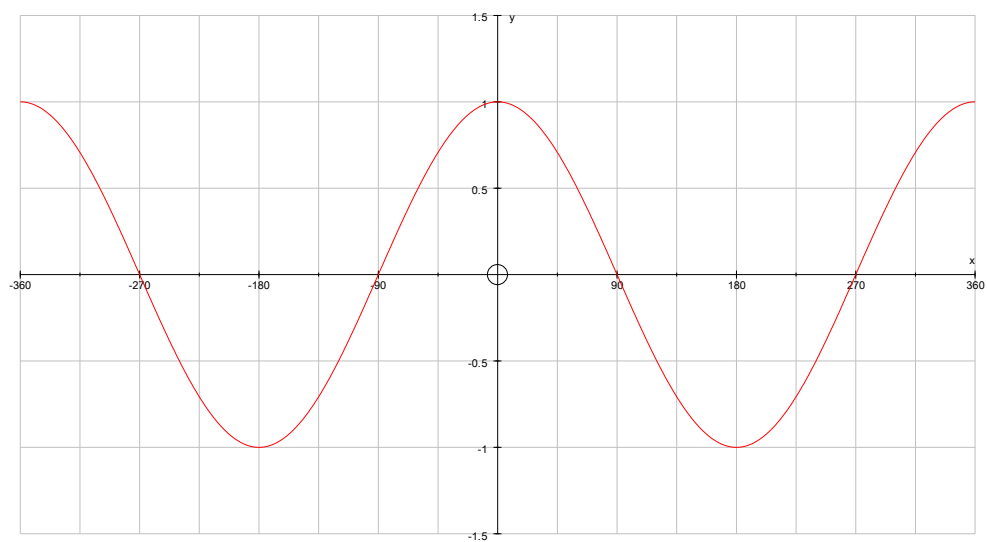
Essential Knowledge

You need to know the shapes of the graphs of the three trigonometric functions and the points at which they cross the axes.

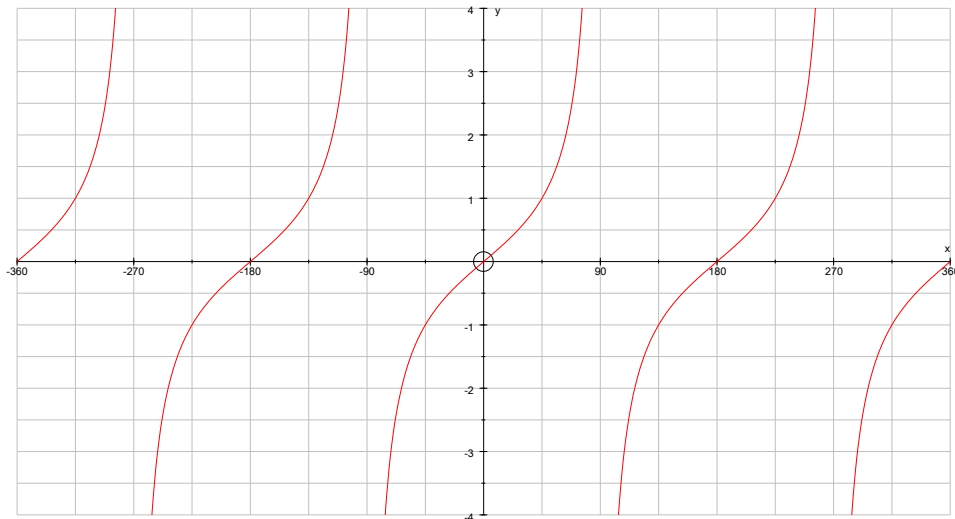
$\sin x$



$\cos x$



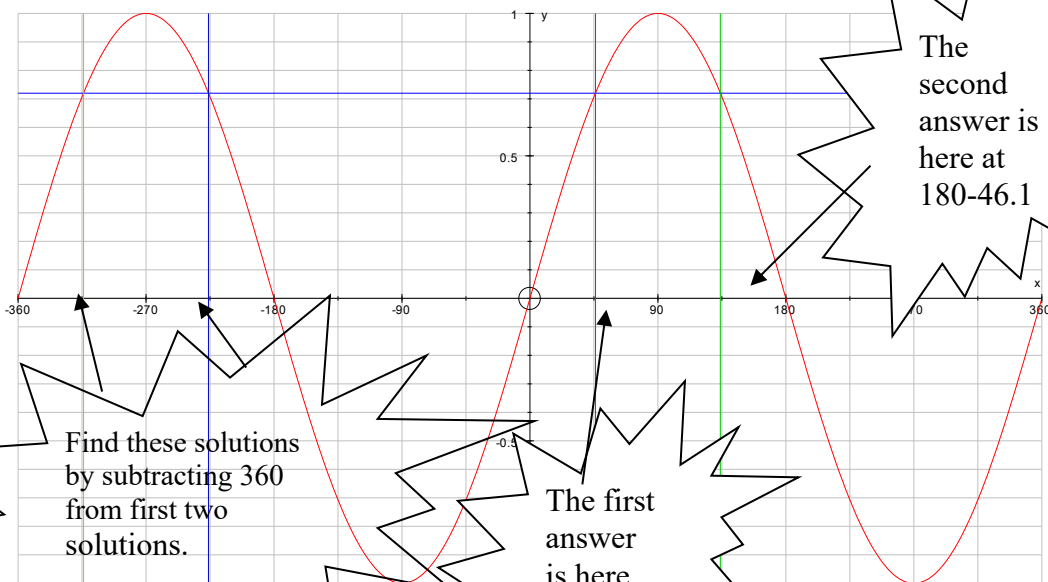
tan x



Using these graphs, you will need to be able to solve equations in a given range.

To solve the equation $\sin x = 0.72$ in the range $-360^\circ \leq x \leq 360^\circ$ find the first solution by typing “SHIFT” “sin” 0.72 “=” into your calculator. (Make sure that your calculator is in degree mode first.)

If you do this you should get the first answer $x = 46.1^\circ$ correct to 3 s.f. Now **sketch** the graph of $\sin x$ in the range specified by the question.



Use the symmetries of the graph to find the other solutions as shown above. So the final answer will be $x = 46.1^\circ, 133.9^\circ, -226.1^\circ, -313.9^\circ$.

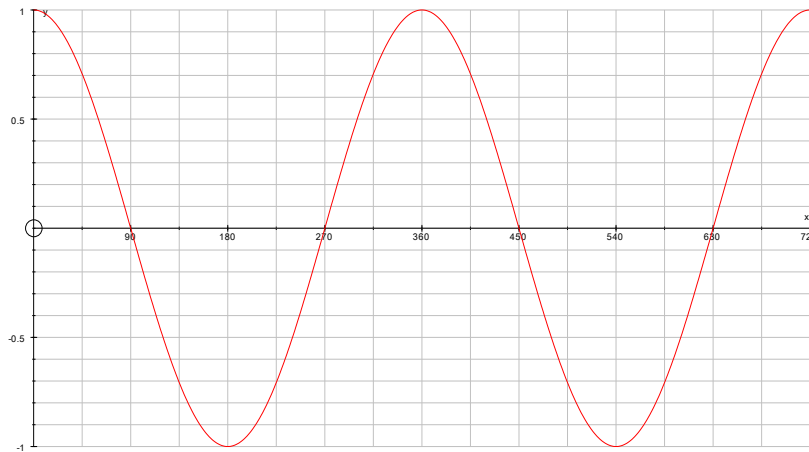
Typical Examination Questions

Question

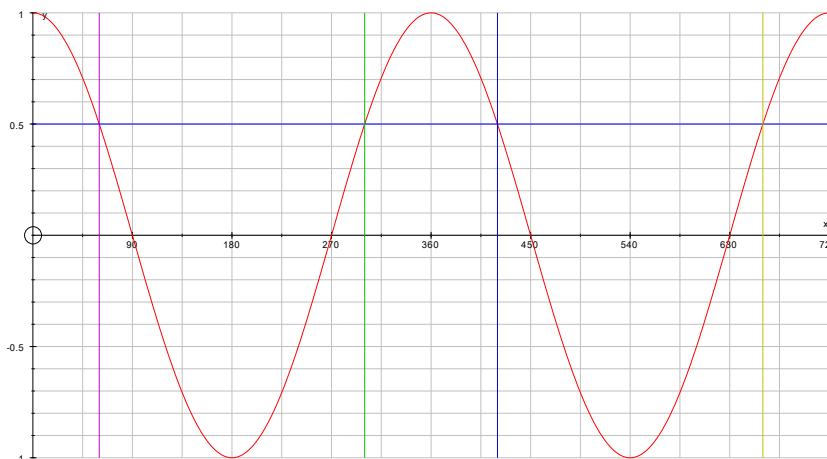
- (a) Sketch the graph of $\cos x$ in the range $0 \leq x \leq 720^\circ$.
- (b) Use your graph to find all of the solutions to the equation $\cos x = 0.5$ in the range $0 \leq x \leq 720^\circ$.

Solution

(a)



(b)



$$x = 60^\circ, 360^\circ - 60^\circ, 60^\circ + 360^\circ, (360^\circ - 60^\circ) + 360^\circ$$

$$\Rightarrow x = 60^\circ, 300^\circ, 420^\circ, 660^\circ$$